WAVE FORMATION IN EXPLOSIVE WELDING

S. K. Godunov, A. A. Deribas,
and N. S. Kozin

A hypothetical analogy between wave formation in explosive welding and the Karman vortex street is analyzed, and the physical laws of the wave formation process are investigated.

In explosive welding periodic waves are usually formed at the contact surface of the colliding metal plates (Fig. 1).

This was first observed by M. A. Lavrent'ev's group in 1944 [1]. Similar wave formation was observed by Allen et al. [2]. Abrahamson [3] has offered a qualitative explanation of the wave formation mechanism. In [4] and elsewhere it has been suggested that there is a parallel between the mechanisms of wave formation as a result of impact and periodic vortex street formation as a result of fluid flow over a cylinder. In [5], Bahrani, Black, and Crossland critically evaluated Abrahamson's ideas and put forward their own qualitative hypotheses concerning the wave formation mechanism, basing their conclusions on the notion of the periodic development and collapse of jets. In [6] an attempt was made to give a quantitative description of wave formation based on the known instability of the tangential velocity discontinuity in an ideal fluid. This attempt cannot be considered successful, since we now know of cases of wave formation in symmetrical collisions, when there are no tangential discontinuities at all.

In the experiments described in [7, 8], a relation was established between the wavelength λ and amplitude a and the collision parameters. When the thickness of the stationary plate δ_2 is many times greater than that of the moving plate δ_1, this relation is given by

$$\frac{\lambda}{\delta_1} = 26 \sin^2 \frac{\gamma}{2} \qquad (\gamma \text{ is the impact angle}) \tag{1.0}$$

Under all impact conditions the wave amplitude a for various metals was of the order of the wavelength

$$a / \lambda \approx 0.25$$

The experiments also showed that wave formation is observed when the velocity U of the high-pressure zone (in the plane case the velocity of the contact point) is less than the speed of sound c_0 in the colliding metals.

The wave excitation process was examined in [9], where the equations of linear acoustics were used to estimate the characteristic dimension of the high-pressure zone. The same paper includes a numerical calculation of the two-dimensional nonstationary collision problem. The computed values of the hydrodynamic quantities are compared with the results following from the linear theory.

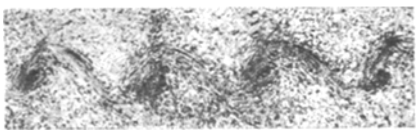

Fig. 1

1. Wave Formation and the Karman Vortex Street. In [4] an analogy was proposed between wave formation in explosive welding and the Karman vortex street formed in a fluid flowing over a cylinder.

We will show that an analysis of the principal factors involved leads to the conclusion that such an analogy is inadmissible.

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Flow over a cylinder is known to create a nonturbulent periodic vortex street on the interval of Reynolds numbers from 20 to 300, while the frequency of eddy formation is determined by a value of the Strouhal number equal to 0.2.

The vortex street behind a cylinder is well described by the Oseen approximation of the Navier–Stokes equations presented in [10]. This is confirmed by comparison with an experiment to calculate the drag force acting on a cylinder in a flow. The vorticity $\Omega(x, y) = u_y - v_x$ of the wake formed behind the cylinder is then given by the following equation:

$$\Omega = \frac{2U\sqrt{\pi U/\nu}}{2\ln(\gamma^* Ua/\nu)-1}\frac{\sin\theta}{\sqrt{r}}e^{k(x-r)}\left(k=\frac{U}{2\nu}\right) \tag{1.1}$$

Here, r and ϑ are polar coordinates, ν is the viscosity of the fluid, a is the cylinder radius, and $\gamma^* = 1.78, \ldots$ is the Euler constant. If the analogy holds, we may assume that the Oseen approximation correctly describes the wake beyond the contact point.

The equations of motion take the form

$$U\frac{\partial u}{\partial x}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$
$$U\frac{\partial v}{\partial x}+\frac{1}{\rho_0}\frac{\partial p}{\partial y}=\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{1.2}$$

Their solution in the functions Φ and Ψ is written in the form

$$u=\frac{\partial\Phi}{\partial x}-\frac{1}{2}e^{kx}\Psi+\frac{1}{2k}\frac{\partial\Psi}{\partial x},\qquad v=\frac{\partial\Phi}{\partial y}+\frac{1}{2k}\frac{\partial\Psi}{\partial y}e^{kx} \tag{1.3}$$

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}=0,\qquad \frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}=k^2\Psi \tag{1.4}$$

The functions Φ and Ψ are selected on the basis of the following conditions: The flow must be symmetrical about the x axis, the wake beyond the contact point must be similar to that behind a cylinder, i.e., the vorticity of the unknown wake must differ only by a factor from (1.1), and in the neighborhood of the contact point x = 0 the asymptotic behavior of the solution must be the same as that for colliding jets obtained in [9].

With these assumptions Φ and Ψ are uniquely defined, and the vorticity Ω has the form

$$\Omega = 2c_0k\frac{\sin\theta}{\sqrt{r}}\cos\frac{\theta}{2}e^{k(x-r)} \tag{1.5}$$

The constant c_0 is found after computing the momentum J of the wake, which can be determined from the first of equations (1.3). On the other hand, it is possible to determine the loss of momentum by considering the jet collision problem. Comparing the expressions obtained, for the vorticity we finally can write

$$\Omega = 2k\frac{\delta_1\delta_2\sin^2(\gamma/2)}{(\delta_1+\delta_2)\pi^{3/2}}\left(\frac{U^3}{\nu}\right)^{1/2}\frac{\sin\theta}{\sqrt{r}}\cos\frac{\theta}{2}e^{k(x-r)} \tag{1.6}$$

Here, δ_2 is the thickness of the bottom plate. Comparing (1.1) and (1.6), we see that

$$\frac{2U\sqrt{\pi U/\nu}}{2\ln(\gamma^* Ua/\nu)-1}=2k\frac{\delta_1\delta_2}{\delta_1+\delta_2}\sin^2\frac{\gamma}{2}\pi^{-3/2}\sqrt{U^3/\nu}$$

Hence we find the radius a of the cylinder, whose wake coincides with that beyond the contact point

$$a=\frac{\nu}{U}\frac{4\sqrt{e}}{\gamma^*}\exp-\frac{4\pi\nu}{U\alpha}\approx 4\frac{-\nu}{U}\exp-\frac{4\pi\nu}{U\alpha}\qquad\left(\alpha=\frac{4}{\pi}\frac{\delta_1\delta_2}{\delta_1+\delta_2}\sin^2\frac{\gamma}{2}\right)$$

The corresponding Reynolds number

$$R=\frac{aU}{\nu}\approx 4\exp\frac{-4\pi\nu}{U\alpha} \tag{1.7}$$

obviously does not exceed 4, which contradicts the experimental data on Karman street formation.

It follows from (1.7) that

$$\ln\frac{4}{R}=\frac{4\pi\nu}{U\alpha}=\frac{4\pi\nu}{aU}\frac{a}{\alpha}=\frac{4\pi}{R}\frac{a}{\alpha}$$

However, this is possibly only if

$$\alpha/a\pi > e = 2.71...$$

On the other hand, the Strouhal number S can be expressed in terms of the wavelength λ

$$S=\frac{1}{2\pi}\frac{\lambda}{a}\qquad\left(S=\frac{U}{\omega a},\ \lambda=\frac{2\pi U}{\omega}\right)$$

Assuming that $\lambda = 20\alpha$, as follows from explosive welding experiments [see Eq. 0.1)], we find that the Strouhal number

$$S\approx\frac{20}{2\pi}\frac{\alpha}{a}>\frac{20}{2}2.72=27.2$$

We know from experiments with vortex layers that S = 0.2. In order to estimate the actual values of the Reynolds and Strouhal numbers in collisions we can use the experimental data on the viscosity of metals in shock compression [11, 12]. In this case the calculations give

$$R\sim 10,\quad S\sim 10^2$$

This discrepancy suggests that the wave formation effects in explosive welding and the formation of Karman vortex streets are physically different phenomena.

2. Propagation of Linear Perturbations in Colliding Plates. By means of the linear equations of acoustics [9], we can investigate the possible modes of linear vibration associated with collision and, in relation to the effect of the free surfaces, their damping.

In [9] the following system of equations was employed:

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+\frac{1}{\rho_0}\frac{\partial p}{\partial x}=0,\qquad \frac{\partial v}{\partial t}+U\frac{\partial v}{\partial x}+\frac{1}{\rho_0}\frac{\partial p}{\partial y}=0 \tag{2.1}$$

$$\frac{\partial p}{\partial t}+U\frac{\partial p}{\partial x}+\rho_0 c_0^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=0$$

Here, U + u, v, and p are the velocity and pressure components, respectively, and ρ_0 is the density of the colliding metals.

In [9] it was also shown that the steady-state solution of (2.1), describing the distribution of the hydrodynamic quantities in the neighborhood of the contact point, has the form

$$u=-a_1\frac{\cos(^1/_2\,\theta)}{\sqrt{r_1}},\quad v=-a_1\frac{\sin(^1/_2\,\theta)}{\sqrt{r_1}},\quad p=a_1\frac{\cos(^1/_2\,\theta)}{\sqrt{r_1}} \tag{2.2}$$

Here,

$$r_1=\left[\left(\frac{x}{\sqrt{1-U^2/c_0^2}}\right)^2+y^2\right]^{1/2},\quad \operatorname{tg}\theta=\frac{y\sqrt{1-U^2/c_0^2}}{x},\quad a_1=1.3\delta_1\sin^2\frac{\gamma}{2}$$

In this solution it was assumed that the impact angle γ is small and that the free surfaces of the plates are represented by the upper and lower ends of the interval x < 0, y = 0. The shape of the free surface η (x) was determined from the equation

$$\frac{\partial\eta}{\partial t}+U\frac{\partial\eta}{\partial x}=v\Big|_{y=0} \tag{2.3}$$

whose solution is the parabola

$$\eta=\sqrt{|x|\,2a_1/U}$$

It is natural to assume that the propagation of periodic perturbations from the neighborhood of the contact point is also described by the solution of system (2.1). Eliminating the velocity components from (2.2), we obtain the equation for the pressure

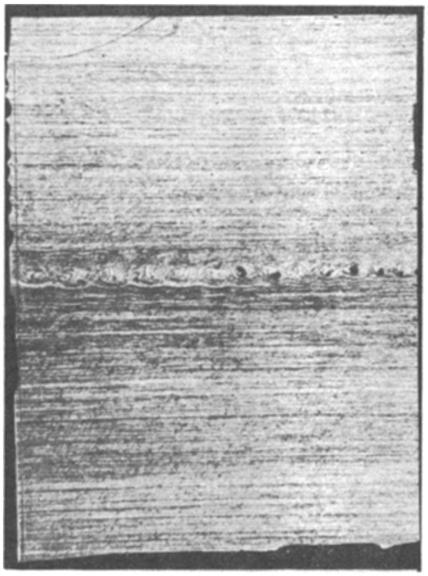

Fig. 2

$$\frac{\partial^2 p}{\partial t^2} + 2U \frac{\partial^2 p}{\partial x\, \partial t} = (c_0{}^2 - U^2)\frac{\partial^2 p}{\partial x^2} + c_0{}^2 \frac{\partial^2 p}{\partial y^2} \tag{2.4}$$

We make the following change of variables:

$$x' = \frac{x - x_0}{\sqrt{1 - U^2 / c_0^2}}, \qquad y' = y.$$
$$t' = \left(1 - \frac{U^2}{c_0^2}\right)^{1/2} t + \frac{U(x - x_0)}{c_0 \sqrt{1 - U^2/c_0^2}} \tag{2.5}$$

Here, x_0 is a certain shift of the coordinate origin. In the new variables Eq. (2.4) takes the form of the usual wave equation

$$\frac{\partial^2 p}{\partial t'^2} = c_0{}^2\left(\frac{\partial^2 p}{\partial x'^2} + \frac{\partial^2 p}{\partial y'^2}\right) \tag{2.6}$$

This equation has a solution, periodic with respect to time, of the form

$$p = A \sin(k_1 c_0 t')\, Z_\nu (k_1 \sqrt{x'^2 + y'^2}) \sin(\nu\theta + \alpha) \qquad (\operatorname{tg}\theta = y'/x') \tag{2.7}$$

Here, Z_ν is a Bessel function, and α and k_1 are arbitrary constants.

Returning to the initial variables, we obtain

$$p = A \sin\left[k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t + \frac{k_1 U}{c_0 \sqrt{1 - U^2/c_0^2}}(x - x_0)\right] \sin(\nu\theta + \alpha)\, Z_\nu(k_1 r_2) \tag{2.8}$$

$$\operatorname{tg}\theta = \frac{y}{x - x_0}\left(1 - \frac{U^2}{c_0^2}\right)^{1/2}, \quad r_2 = \left[\frac{(x - x_0)^2}{\sqrt{1 - U^2/c_0^2}} + y^2\right]^{1/2} \tag{2.9}$$

From (2.8) and (2.1) for the velocity components u and v we obtain

$$\begin{aligned} u &= u_c \cos(k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t) + u_s \sin(k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t) \\ v &= v_c \cos(k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t) + v_s \sin(k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t) \end{aligned} \tag{2.10}$$

Here, u_c, u_s, v_c, and v_s are functions of the coordinates x and y determined from the first two equations of system (2.1).

Confining our attention to waves going to infinity, in (2.8) we can set

$$Z_\nu(kr) = H_\nu^{(1)}(kr)$$

Here, $H_\nu^{(1)}(kr)$ is a Hankel function of the first kind of order ν. We assume that the source of the perturbations is located at a certain point x_0 in the high-pressure zone near the contact point. The dimensions of this zone were determined in [9] and we shall assume that

$$x_0 \sim \delta_1 \sin^2 \gamma/2 \tag{2.11}$$

The order of the Hankel function ν and the phase shift α are determined by the following considerations. The boundary condition $p = 0$ at the free edge $x < 0$, $y = 0$ can be satisfied by setting $\nu = 1/2, 1, 3/2, 2, \ldots$ and selecting α equal to 0 at integral values of ν and equal to $\pi/2$ at fractional values of ν. Having determined u_c, u_s, v_c, v_s from (2.1) and using (2.10), from (2.3) we can find the wave form at the free surface.

We note that in (2.8) the asymmetrical waves usually observed correspond to the case $\nu = 1$. In individual cases, by ensuring the precise symmetry of the collision conditions, it is possible to observe symmetrical waves (Fig. 2) corresponding to $\nu = 1/2$. The possibility of the occurrence of higher harmonics, corresponding to higher modes of symmetry, is not yet clear.

Thus, it is natural to conclude that in most experiments on wave formation the values of the hydrodynamic quantities are determined by the pressure equation

$$p = A \sin\left[k_1 c_0 \sqrt{1 - U^2/c_0{}^2}\, t + \frac{k_1 U (x - x_0)}{c_0 \sqrt{1 - U^2/c_0^2}}\right] H_1^{(1)}(k_1 r) \sin\theta \tag{2.12}$$

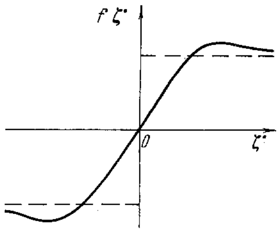

Fig. 3

and relations (2.10) for the velocities. Obviously, in the actual process the source is not concentrated in the point x_0 but distributed over a certain small volume in the neighborhood of the contact point.

The oscillations described by Eq. (2.12) reflect the actual wave process only at distances from the contact point considerably exceeding the dimension of the high-pressure zone $\delta_1 \sin^2(\gamma/2)$ and less than the thickness of the moving plate δ_1. At greater distances from the contact point the effect of the free edges leads to an exponential decline in amplitude.

We will illustrate the "freezing" process with the following elementary example. Consider a source of harmonic oscillations moving to the left along the x axis with velocity U, the distribution of the velocity component along the y axis at y = 0 having the following form:

$$v = \sin \omega t \, e^{-(x+Ut)}$$

From the equation for the displacement of the free surface (2.3) we have

$$\frac{\partial \eta}{\partial t} = v \,|_{y=0} = \sin \omega t \, e^{-(x+Ut)}$$

Hence we find the free surface at x + Ut > 0 (behind the source)

$$\eta = A_0 \sin(\omega x/U) + \sin \omega t \, e^{-(x+Ut)}$$

Here, A_0 is the amplitude of the waves at the contact surface for sufficiently large t. The equation obtained shows that, after passage of the source through the point x, in the course of time the shape of the boundary there is frozen and takes the form

$$\eta = A_0 \sin(\omega x/U)$$

3. Explosive-Welding Waves from the Self-Oscillation Standpoint. 1. As we know from the general theory of self-oscillations (see, for example, [13]), any self-oscillatory system must contain an oscillator with a certain frequency ω and damping constant $\omega/2\pi\tau_0$. Moreover, there must be an energy pumping mechanism, which in actual systems is usually related with the presence of a nonlinear "negative" frictional resistance. The equation describing this process in a system with one degree of freedom has the form

$$\zeta^{\cdot\cdot} + \frac{2}{\tau_0}\zeta^{\cdot} + \left(\omega^2 + \frac{1}{\tau_0^2}\right)\zeta = f(\zeta^{\cdot}) \tag{3.1}$$

Here, ζ is the deviation of the system from the equilibrium position. A typical graph of the function $f(\zeta^{\cdot})$ for a system with so-called "hard" excitation is presented in Fig. 3. The $f(\zeta^{\cdot})$ curve can be approximately represented in the form of a piecewise-constant function (Fig. 3). Obviously, given a weak initial perturbation, in such a system with hard excitation, oscillations do not occur, since the system does not fall within the interval of nonzero values of f.

Assuming that the self-oscillation frequency is approximately equal to ω and that the oscillations of the point are sinusoidal

$$\zeta = B \sin \omega (t - t_0) \tag{3.2}$$

we can approximately determine their amplitude from energy considerations, multiplying Eq. (3.1) by $\zeta^{\cdot}$ and integrating over the oscillation period. We have

$$0 = \left[\frac{\zeta^{\cdot 2}}{2} + \omega^2\zeta^2\right]_{t_0}^{T} = \int_{t_0}^{T}\left[-\frac{\zeta^{\cdot 2}}{\tau_0} + \zeta^{\cdot} f(\zeta^{\cdot})\right] dt = -\frac{\omega^2 B^2}{2\tau_0} + \int_0^{2\pi} \omega B \cos\theta f(\omega B \cos\theta)\, d\theta \approx -\frac{\omega^2 B^2}{2\tau_0} + K 2 F \omega B \tag{3.3}$$

$$K = K(B) \leqslant 1 \qquad (T = t_0 + 2/\pi\omega)$$

The factor K is present owing to the fact that $f(\zeta^{\cdot}) = 0$ at $|\zeta^{\cdot}| < \zeta_0^{\cdot}$. At large B the coefficient K may be assumed equal to unity. From (3.3) we obtain

$$B = \frac{4F\tau_0}{\omega} K \approx \frac{4F\tau_0}{\omega}$$

We note that a decrease in $f'(\zeta^{\cdot})$ at large $\zeta^{\cdot}$ is necessary for the stability of the limit cycle ensuring the self-oscillatory regime [13].

2. We will consider the case when $f(\zeta^{\cdot})$ is piecewise-linear and the following relations are satisfied:

$$f'(\zeta^{\cdot}) = F_1 \qquad (\zeta^{\cdot} < \zeta_0^{\cdot})$$
$$f'(\zeta^{\cdot}) = F_2 > F_1 \qquad (\zeta_0^{\cdot} < \zeta^{\cdot} < \zeta_1^{\cdot})$$
$$f'(\zeta^{\cdot}) = F_3 > F_2 \qquad (\zeta_1^{\cdot} < \zeta^{\cdot})$$

In this case, if

$$F_1 < \frac{1}{2\tau_0}, \qquad F_3 < \frac{1}{2\tau_0}, \qquad F_2 > \frac{1}{2\tau_0}, \qquad \zeta_1^{\cdot} < \zeta_0^{\cdot}$$

and $\zeta_0^{\cdot}$ is sufficiently large, self-oscillations analogous to the previously considered case are possible in the system. It is easy to see that the equation

$$-\frac{\omega^2 B^2}{2\tau_0} + \int_0^{2\pi} \omega B \cos\theta f(\omega B \cos\theta)\, d\theta = 0$$

has a unique solution. Other nonlinear dependences $f(\zeta)$ and even $f(\zeta^{\cdot}, \zeta)$, leading to the appearance of self-oscillations, are also possible.

3. In [9], as a result of an analysis of the experimental data and the numerical and analytic solutions of the equations of hydrodynamics it was established that wave formation requires a sufficiently strong initial perturbation of the collision process. The expansion wave reaching the contact point from the free surface of the moving plate was also investigated.

In addition, special experiments, in which perturbations were artificially created – by projections on the stationary plate – were also performed. These investigations led to the conclusion that wave formation in explosive welding is not a manifestation of some instability, being a self-oscillatory process with hard excitation whose mechanism is concentrated in a small neighborhood of the contact point.

In order to apply to the analysis of this process the facts derived above from the theory of oscillations, it is necessary to determine what must be understood by the deviation ζ and the oscillator characteristics ω and τ_0 and to establish the nature of the collision processes corresponding to the dependence $f(\zeta^{\cdot})$.

4. The Oscillator. In [9] it was shown with reference to the hydrodynamic collision model that the density and temperature of the colliding plates differ substantially from their starting values in a neighborhood of the contact point with linear dimension

$$R^{\circ} = \frac{2}{\pi}\left(1 - \frac{U^2}{c_0^2}\right)^{1/2} \frac{2\delta_1\delta_2}{(\delta_1 + \delta_2)} \sin^2\frac{\gamma}{2} \tag{4.1}$$

In this neighborhood the speed of sound c also differs substantially from the starting value c_0 and the region may be regarded as an oscillator with frequency ω of the order of $c_0 R^{\circ}$, i.e.,

$$\omega \approx c_0 R^{\circ}$$

In the oscillation process waves are radiated into the surrounding medium; these waves are damped and the characteristic damping time τ may be assumed to be of the order of the period

$$\tau \approx 2\pi / c_0 R^{\circ}$$

We will consider an example of such an oscillator in the hydrodynamic approximation. It may be assumed that consideration of the analogous elastic problems will not produce significant changes in the character and order of magnitude of the frequency and damping constant.

1. The complex oscillator frequencies $\varkappa = i\omega - \tau^{-1}$ can be determined as the eigenvalues of the following system:

$$\varkappa u + U(x, y)\frac{\partial u}{\partial x} + V(x, y)\frac{\partial u}{\partial y} + \frac{1}{\rho(x, y)}\frac{\partial p}{\partial x} = 0$$

$$\varkappa v + U(x, y)\frac{\partial v}{\partial x} + V(x, y)\frac{\partial v}{\partial y} + \frac{1}{\rho(x, y)}\frac{\partial p}{\partial y} = 0 \tag{4.2}$$

$$\varkappa p + U(x, y)\frac{\partial p}{\partial x} + V(x, y)\frac{\partial p}{\partial y} + \rho(x, y)c^2(x, y)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

The velocities U(x, y), V(x, y), the density distribution ρ(x, y), and the square of the speed of sound c^2(x, y) must be found from the solution of the steady-state problem, a linear model of which was described in Sec. 2. In the general case it is difficult to find the eigenvalues of this system, and the task involves laborious machine computations. Below we present a rough calculation of the system for the case U = 0, V = 0. We note that usually the collision conditions are such that $(U^2 + V^2)/c^2 < 1/4$.

We also schematize the distribution of the density and speed of sound, setting

$$\rho(x, y) = \begin{cases} \rho_0 & (x^2 + y^2 > R^{\circ 2}) \\ \rho^* > \rho_0 & (x^2 + y^2 < R^{\circ 2}) \end{cases} \tag{4.3}$$

$$c(x, y) = \begin{cases} c_0 & (x^2 + y^2 > R^{\circ 2}) \\ c^* > c_0 & (x^2 + y^2 < R^{\circ 2}) \end{cases}$$

Since the pressure is equal to zero on the interval x < 0, y = 0, we find the solution of system (4.1) in the form

$$p(x, y) = \sin\theta\, W(r) \tag{4.4}$$

Thus, only antisymmetric modes are investigated. Substituting (4.4) into (4.2), we obtain

$$p = \sin\theta Z_1\left(\frac{i\varkappa}{c}r\right) \qquad \left(c = \begin{cases} c_0 & (r > R^\circ) \\ c^* & (r < R^\circ)\end{cases}\right) \tag{4.5}$$

We note that here Z_1 stands for the different branches of cylindrical functions of the first order for r less than and greater than R°. For the velocity components we obtain

$$u = -\frac{i\sin 2\theta}{2\varkappa\rho}\left\{-\frac{i\varkappa}{c}Z_0\left(\frac{i\varkappa}{c}r\right) + \frac{2}{r}Z_1\left(\frac{i\varkappa}{c}r\right)\right\} \tag{4.6}$$

$$v = \frac{i}{\rho\varkappa}\left\{\frac{i\varkappa}{2c}Z_0\left(\frac{i\varkappa}{c}r\right) - \cos 2\theta\left[\frac{i\varkappa}{2c}Z_0\left(\frac{i\varkappa}{c}r\right) + \frac{1}{r}Z_1\left(\frac{i\varkappa}{c}r\right)\right]\right\}$$

The condition at infinity (radiation condition) consists in that for r > R°, W(r) coincides correct to an arbitrary constant with the function $H_1^{(1)}(i\varkappa r/c_0)$ (see Sec. 2). At r < R° the regularity at the point x = 0, y = 0 leads to the equation

$$W(r) = \text{const}\, J_1\left(\frac{i\varkappa}{c_0}r\right)$$

The boundary conditions at r = R° (continuity of p, u, and v) can be satisfied only for a discrete series of eigenvalues $\varkappa$. We present the values of $\omega R/c_0$, $R/\tau c_0$ for a number of values of the ratio $\rho_0^* c_0^{*2}/\rho_0 c_0^2$

$\rho_0{}^*c^*{}_0{}^2/\rho_0 c_0{}^2 =$	2.25	3.38	4.00	4.50	6.00	8.00
$\omega R^\circ/c_0 =$	1.80	1.83	1.82	1.85	1.85	1.86
$-R^\circ/\tau c_0 =$	0.38	0.42	0.40	0.40	0.44	0.45

From these data it follows that the damping constant $R/\tau c_0$ and the frequency $\omega R^\circ/c_0$ depend only weakly on the parameter $\rho_0{}^* c_0{}^{*2}/\rho_0 c_0{}^2$. In this case, $R^\circ/\tau c_0 \approx -0.4$, $\omega R^\circ/c_0 \approx 2$, i.e., the frequency is of the order of the width of the high-pressure zone.

5. Energy Pumping Mechanism. The existence of undamped self-oscillations presupposes that energy is pumped in in the neighborhood of the contact point. We will attempt to describe a possible variant of the energy pumping mechanism. We start with the system of equations of hydrodynamics with the viscous stress tensor

$$\frac{\partial\rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(p + \rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = \frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} \tag{5.1}$$

$$\frac{\partial(\rho v)}{\partial t}+\frac{\partial(\rho uv)}{\partial x}+\frac{\partial(p+\rho v^2)}{\partial y}=\frac{\partial\sigma_{21}}{\partial x}+\frac{\partial\sigma_{22}}{\partial y}$$

$$\rho T\left(\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}\right)=u_x\sigma_{11}+(u_y+v_x)\sigma_{12}+v_y\sigma_{22}$$

Here, S is the entropy density per unit mass, and the pressure p is a function of density and entropy, $p = p(\rho, S)$.

The right side of the last equation determines the heat released in internal friction. It is natural to assume that the total strength of the heat source is a function of the strain rate distribution.

From system (5.1) we obtain the equation for the pressure

$$\frac{d}{dt}\left(\frac{1}{\rho c^2}\frac{dp}{dt}\right)-\frac{\partial}{\partial x}\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right)=\frac{d}{dt}\left(\frac{p_S}{\rho^2 Tc^2}Q\right) \qquad (5.2)$$

$$\left(\frac{d}{dt}=\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)$$

If we assume that the function Q(x, y, t) is nonzero in a certain neighborhood of the contact point and has the character of a dipole, i.e., postulate nonsymmetrical heat release at $y > 0$ and $y < 0$, we may conclude that the existence of self-oscillations requires that the right side of Eq. (5.2) may, with sufficient accuracy, be regarded as a dipole of strength f approximately represented by a nonlinear function of $d\langle p\rangle/dt$. Here, $\langle p\rangle$ is the mean pressure in one half of the neighborhood. (The pressure distribution must also be assumed to be dipole in character.)

For self-oscillations to exist the nature of the dependence $f\,(d\langle p\rangle/dt)$ must be nonlinear, as shown in Sec. 3.2. In all probability, the nonlinearity of this dependence is a consequence of the plasticity of the material. In fact, the stress tensor $\{\sigma_{ik}\}$ must be related with the strain rate tensor, as follows:

$$\begin{pmatrix}\sigma_{11} & \sigma_{12}\\ \sigma_{21} & \sigma_{22}\end{pmatrix}=\eta\begin{pmatrix}u_x-v_y & u_y+v_x\\ u_y+v_x & u_x-v_y\end{pmatrix}+\left(\zeta+\frac{\eta}{3}\right)\begin{pmatrix}u_x+v_y & 0\\ 0 & u_x+v_y\end{pmatrix} \qquad (5.3)$$

The Mises yield condition [14] can be understood as the following relation between η and the strain rates:

$$\eta=\begin{cases}\eta_0 & (\Delta<Y_0)\\ \eta_0 Y_0/\Delta & (\Delta>Y_0)\end{cases} \qquad (5.4)$$

$$\Delta=\sqrt{(u_x-v_y)^2+(u_y+v_x)^2}$$

We assume that in a certain region, modeled in Sec. 2 by a singularity of the Bessel function, harmonic oscillations are taking place according to the law

$$u(x,y,t)=u_0(x,y)+D\sin\omega t\,u_1(x,y)$$
$$v(x,y,t)=v_0(x,y)+D\sin\omega t\,v_1(x,y) \qquad (5.5)$$

For simplicity we further assume that $u_1 \ll u_0$, $v_1 \ll v_0$. In the region in question the shear stresses also depend harmonically on time. As a result of the nonlinear relation between η and the shear stresses, the time dependence of Q is not harmonic. It is natural to assume that the energy pumping mechanism is described precisely by a nonlinearity of this type.

Generally speaking, other types of nonlinear relations between the viscous stresses, strain rates, and temperature are possible. To establish these dependences it will be necessary to make a detailed investigation of the behavior of metals at high dynamic pressures.

It is possible that in a thin layer near the contact surface some mechanism similar to Coulomb friction, capable of pumping in the energy required for the existence of self-oscillation, plays an important role.

LITERATURE CITED

1. A. A. Deribas, V. M. Kudinov, F. I. Matveenkov, and V. A. Simonov, "Explosive welding," Fiz. Goreniya i Vzryva, No. 1 (1967).
2. W. A. Allen, T. M. Mapes, and W. G. Wilson, "An effect produced by oblique impact of a cylinder on a thin target," J. Appl. Phys., 25, No. 5 (1964).
3. H. Abrahamson, "Residual periodic surface deformations produced by a traveling jet," J. Appl. Mech., No. 4 (1961).

4. E. Schmidtman, W. Koch, and H. Schenck, "Vorgänge beim Explosivschweissen metallischen Werkstoffe," Arch. Eisenhüttenwesen, 36, No. 9 (1965).
5. A. S. Bahrani, T. J. Black, and B. Crossland, "The mechanics of wave formation in explosive welding," Proc. Roy. Soc., A296, No. 1445 (1967).
6. J. H. Hunt, "Wave formation in explosive welding," Philos. Mag., 17, No. 148 (1968).
7. A. A. Deribas, V. M. Kudinov, and F. I. Matveenkov, "Effect of initial parameters on the process of wave formation in explosive welding," Fiz. Goreniya i Vzryva, No. 4 (1967).
8. A. A. Deribas, V. M. Kudinov, F. I. Matveenkov, and V. A. Simonov, "Modeling the process of wave formation in explosive welding," Fiz. Goreniya i Vzryva, No. 1 (1968).
9. S. K. Godunov, A. A. Deribas, A. V. Zabrodin, and N. S. Kozin, "Hydrodynamic effects in colliding solids," J. Comput. Phys., 5, No. 3 (1970).
10. N. E. Kochin, I. A. Kibel', and N. V. Roze, Theoretical Hydrodynamics, Vol. 2 [in Russian], Gostekhizdat, Moscow-Leningrad (1948).
11. V. N. Mineev and E. V. Savinov, "Viscosity and melting point of aluminum, lead, and sodium chloride under shock compression," Zh. Éksp. i Teor. Fiz., 52, No. 3 (1967).
12. S. K. Godunov, A. A. Deribas, I. D. Zakharenko, and V. I. Mali, "Viscosity of metals in high-velocity collisions," Fiz. Goreniya i Vzryva, No. 1 (1971).
13. A. A. Kharkevich, Self-Oscillations [in Russian], Gostekhizdat, Moscow (1953).
14. V. V. Sokolovskii, Theory of Plasticity [in Russian], Vysshaya Shkola, Moscow (1969).